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# Signaling in OTC Markets: Benefits and Costs of Transparency

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## Abstract

We provide a theoretical rationale for dealer objections to ex post transparency in over-the-counter markets. Disclosure of the terms of a transaction conveys information possessed by the dealer about the asset quality and reduces the dealer's rents when she disposes of the inventory in a second transaction. We show that costly signaling in a transparent market benefits investors through lower spreads and higher volume. Dealers may also gain from transparency despite lower spreads when potential gains from trade are small or adverse selection is high, because in those circumstances higher volume offsets smaller spreads for dealer profits.

## I. Introduction

There has been an increase in ex post transparency in securities markets over time, often over the objections of dealers. This was especially true during the implementation of the Trade Reporting And Compliance Engine (TRACE) system for U.S. corporate bond reporting by the National Association of Securities Dealers (NASD).[1] We investigate the benefits and costs of ex post transparency for investors and dealers in a theoretical model of over-the-counter (OTC) markets.

We consider a dealer who acquires and then disposes of inventory in a sequence of transactions. With ex post transparency, the terms of the first trade may reveal private information of the dealer to the counterparty in the

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[1]The TRACE system was implemented in 2002 by the NASD and expanded in stages through 2005. The reporting requirements are now imposed by the Financial Industry Regulatory Authority. Bravo (2003), Laughlin (2005), Patterson and Zeng (2010), and Wigglesworth (2015) all provide reports of U.S. corporate bond dealers opposing trade disclosure. Similarly, Rothwell (2006) reports European bond dealers opposing disclosure, and Burne (2010) reports swap dealers opposing disclosure.

second transaction. This reduces the dealer's ability to extract rents in the second transaction—the round-trip spread the dealer earns is reduced. Thus, we provide a rationale for dealer objections to ex post transparency. However, we also show that ex post transparency increases both the volume of trade and allocative efficiency. Thus, investors benefit from transparency. In fact, it is even possible for dealers to benefit from transparency, depending on whether the larger volume or the smaller spreads dominate. We determine the conditions under which one or the other is more important.

The mechanism that drives our results is costly signaling. Suppose that trade is initiated by a seller, and after acquiring inventory the dealer needs to find a buyer. We assume there are many investors in the dealer's Rolodex who hold assets of the type being traded but have no special information about this particular asset. The dealer contacts one of these investors and quotes an ask price for the asset. In a transparent market, the price at which the dealer acquired the asset may signal private information possessed by the dealer to the buyer. We consider three types of markets, described as follows: A hypothetical ("full information") market in which the dealer's private information is directly observed by the buyer, a transparent market in which the price of the first transaction is observed by the buyer and hence may signal the dealer's private information to the buyer, and an opaque market in which the buyer has no information other than the ask price quoted by the dealer for the second transaction. Our interest is in comparing the transparent and opaque markets. We show that in a separating equilibrium of the transparent market, dealers "overbid" for the asset in the first transaction to separate in the second transaction from other types of dealers with lower valuations. This overbidding is costly to dealers relative to the full information environment. However, overbidding also increases the likelihood of the dealer's bid being accepted by the seller in the first transaction. Hence, costly signaling increases the volume of trade and increases gains from trade. In fact, realized gains from trade are higher in the separating equilibrium of the transparent market than they are either in the full information environment or in an opaque market.[2]

On a per-trade basis, dealers always prefer opacity. The overbidding in the separating equilibrium of the transparent market causes spreads to be smaller in transparent than in opaque markets. However, the higher volume in transparent markets can sometimes offset the smaller spreads. Thus, dealers may earn higher profits in a transparent market than in an opaque market. Whether they prefer transparent or opaque markets depends on the magnitude of potential gains from trade relative to the extent of adverse selection. Dealers prefer opacity when potential gains from trade are large relative to adverse selection. The potential gains from trade depend on the private motives for trading of the customer who initiates the transactions. When the customer is highly motivated (or when adverse selection is low), trade is highly probable in both transparent and opaque markets. In that case, the increase in the volume of trade due to transparency is not large

[2]We denote "full information" as the environment in which the dealer and second counterparty have the same information. There is still an informational friction, because the first counterparty has private information. Thus, we do not achieve the first-best outcome in our "full information" environment.

enough to overcome the lower spreads, and dealers prefer opacity. On the other hand, if potential gains from trade are low or adverse selection is high, then dealers benefit from transparency. The opposition to transparency cited in footnote 1 and the empirical evidence that bond dealers lost revenues when TRACE was introduced (Bessembinder, Maxwell, and Venkataraman (2006), Edwards, Harris, and Piwowar (2007)) indicate that potential gains from trade were large in those markets relative to adverse selection.

In our model, there is only a single quantity traded, and we study price disclosure. In reality, disclosure involves both quantity and price. We show that dealers dislike price disclosure when gains from trade are large or adverse selection is low. Dealers may also dislike quantity disclosure. However, price disclosure seems to be important, both in our model and in reality. Laughlin (2005) quotes a high-yield bond trader (cited in Bessembinder and Maxwell (2008)) describing the pre-TRACE corporate bond market as "customers felt like they never really knew where a dealer was buying or selling, and they were scared that dealers were working for too much margin.... Many investors now think the real benefit of TRACE lies in knowing that they are not being raked over coals."

Ex post transparency (mandatory disclosure) matters, because investors cannot rely on voluntary disclosure by dealers to be truthful. The usual unraveling argument implies that there would be full and truthful disclosure in equilibrium if false disclosures could be sufficiently punished (Grossman (1981), Milgrom (1981)). However, it may be difficult to detect false statements by dealers, and there may be limited means for punishing false statements. In the United States, mortgage-backed securities dealers from Jefferies, Cantor Fitzgerald, and Nomura were recently prosecuted for lying to investors about the prices at which they had previously transacted. All were either found not guilty or their convictions were reversed on appeal (Dolmetsch (2018)). In his ruling on the appeal of the Nomura case (U.S. v. Shapiro and Gramins (2018)), U.S. District Judge Robert Chatigny stated: "As the Government concedes, lying in arms-length commercial transactions is not always illegal. It depends on the particular facts and circumstances ...Prior to the indictment in Litvak, the conduct at issue appears to have been widespread in the RMBS market. Cooperating witnesses in this case testified that they didn't realize the conduct was illegal." This view that dealers frequently attempt to mislead investors about market prices when disclosure is not mandatory is supported by Reid Muoio, deputy head of the SECs structured and new products unit, who states that false disclosure is "fairly commonplace, fairly widespread" in opaque markets (Faux and Dolmetsch (2013)). Likewise, Levine (2018) provides anecdotal evidence of lying by dealers in the bond market, stating that "for years bond traders lied to customers about the prices they had paid."

We assume that the investor who is motivated to trade contacts only a single dealer, and the dealer has all of the bargaining power in the transaction. In particular, we assume the dealer makes the investor a take-it-or-leave-it offer, which is common in OTC markets. According to Duffie (2012), "An OTC trade negotiation is typically initiated when an investor contacts a dealer and asks for terms of trade ...A dealer making two-sided markets typically provides a take-it-or-leave-it pair of prices, a bid and an offer, to a customer." In actual markets, there are of course multiple dealers and a customer can reject an offer from one dealer and seek a better offer from another. However, if search is costly, then the outcome is the same as if there were only one dealer. With costly search, the Diamond paradox (Diamond (1971)) states that each dealer will behave as a monopolist, because the trader's only alternative to trading with the dealer with whom he is currently in contact is to incur the search cost and then contact another dealer who in equilibrium also behaves as a monopolist. What a trader cannot do in an OTC market is to solicit quotes from multiple dealers and then select the best one; that is, he cannot induce dealers to engage in Bertrand competition. According to Bessembinder and Maxwell (2008), "Dealer quotations in corporate bonds are not disseminated broadly or continuously. Quotations are generally available only to institutional traders, mainly in response to phone requests ... Telephone quotations indicate a firm price but are only good 'as long as the breath is warm,' which limits one's ability to obtain multiple quotations before committing to trade." Bessembinder and Maxwell go on to point out that even in OTC markets with electronic messaging systems, "price quotations mainly serve as an indication of the desire to trade, not a firm obligation on price and quantity." In fact, whether competition is sequential or simultaneous would seem to be the key distinction between OTC markets and exchanges. Even if search is costless, sequential competition is not the same as simultaneous competition. In a theoretical model of informed dealers, Zhu (2012) shows that a trader who turns down a dealer's quote and then returns to that dealer after seeking other quotes will be disadvantaged. This is because the dealer can infer that other dealers have offered worse quotes and hence must have worse signals. This implies that the dealer will offer worse terms when contacted a second time. Thus, costless sequential search is not equivalent to generating multiple quotes and choosing the best one; that is, it is not equivalent to running an auction among dealers.

We also assume the dealer makes a take-it-or-leave-it offer in the second transaction that serves to offload the dealer's inventory. This assumption is motivated by the prevalence of take-it-or-leave-it offers in actual OTC markets. However, we could dispense with the assumption in the second transaction and assume that the dealer instead runs an auction among possible traders. Given our assumption that the traders in the second transaction are uninformed about the asset quality, this auction will be a Bertrand game in which the traders compete away all rents. The resulting transaction price will be exactly the price at which the asset trades in our model with take-it-or-leave-it dealer quotes.

For some assets, interdealer markets are important. They provide an alternative way for dealers to offload inventory rather than trading with a second customer. Costly signaling should also be important in the presence of an interdealer market if the original dealer has information not possessed by other dealers.

The remainder of this article is organized as follows: The next section provides a further survey of the literature. Section III presents the model and describes the full information equilibrium. Section IV analyzes the transparent market, and Section V analyzes the opaque market. Section VI compares the three market types numerically under more specific distributional assumptions (e.g., a normally distributed error term in the dealer's signal). Section VII concludes the article.

## II. Literature Review

Signaling by intermediaries is a relevant issue due to the importance and the distinct nature of OTC markets, but we are not aware of any other paper on this topic. There are many studies on the role of signaling in dynamic asset markets (see Kremer and Skrzypacz (2007), Daley and Green (2012), Fuchs and Skrzypacz (2013), Kurlat (2013), Guerrieri and Shimer (2014), Fuchs, Öry, and Skrzypacz (2016), and references therein). However, the market design question related to the effect of transparency on welfare is absent from this literature with the exception of Fuchs et al. They study a seller's ability to signal a high asset value by rejecting offers and show that transparency, in the form of disclosing all price offers, reduces the probability of trading. We reach the opposite conclusion because, consistent with actual disclosure requirements, we study disclosure of transaction prices instead of disclosure of rejected offers.

There exists research on the welfare effects of transparency in OTC markets. The paper closest to ours is Naik, Neuberger, and Viswanathan (1999). Like us, they study a dealer who first trades with an informed customer and then offloads the inventory in a second transaction. They conclude that transparency can reduce the welfare of investors, depending on how much the dealer learns about the asset value in making the first transaction. A key difference between their paper and ours is that they assume dealers are competitive and do not earn any rents, in either a transparent or opaque market. Thus, dealers should be indifferent about transparency under their assumptions. This runs counter to the evidence cited in footnote 1 about dealers opposing disclosure and to the empirical evidence that bond dealers did lose revenues following the introduction of TRACE (Bessembinder et al. (2006), Edwards et al. (2007)). Bessembinder and Maxwell (2008) provide interesting anecdotal evidence that dealers suffered from the introduction of TRACE. A second difference between our paper and Naik et al. is that the dealer in their model is initially uninformed, so the key mechanism that we study (quoting excessively favorable prices in the first transaction to separate from dealers with worse signals) does not arise in their model.

Other papers that study the welfare implications of various forms of transparency in OTC markets include Cujean and Praz (2015). They show that better information about counterparties' liquidity needs improves aggregate welfare through improved allocative efficiency in an interdealer market. Dealers are absent in their model; thus, they do not address the effects of transparency on dealers' welfare. Bhattacharya (2016) assumes that dealers compete in an auction to trade with a customer and may run an auction after the transaction to trade among themselves. He is not able to solve for the equilibrium of his dynamic model explicitly, but he is able to compare some features of equilibrium strategies with and without transparency. Duffie, Dworczak, and Zhu (2017) show that the publication of benchmark prices in OTC markets improves market efficiency while reducing per-trade profitability for the dealers. They find that the net effect is often an increase in aggregate welfare, because the publication of benchmarks reduces the information advantage of dealers over customers. Asriyan, Fuchs, and Green (2017) examine the spillover effects of transparency when investors in distinct markets can observe trades in correlated assets.

Most models of securities dealers (e.g., Glosten and Milgrom (1985), Kyle (1985)) assume dealers are uninformed. There are some exceptions, but the exceptions do not study signaling by informed dealers. Glode and Opp (2016) study intermediation between an uninformed seller and an informed buyer via a chain of increasingly informed dealers. They assume that the less informed party makes the offer in each bilateral trade, so there is no possibility of signaling. Endogenous signaling does not arise in Duffie et al. (2017) because the market is assumed to be opaque.

Our assumption that a dealer searches for a second counterparty to offload inventory is shared by Rubinstein and Wolinsky (1987) and Duffie, Gârleanu, and Pedersen (2005). However, in those papers, search is modeled by random matching, and all agents have identical information. The presence of dealers in those models reduces search time for customers who have private motives for trading, and some of the welfare gain is captured by dealers. In our model, search for the second counterparty is costless for the dealer.

Some of our model's predictions are supported by empirical evidence. Edwards et al. (2007), Bessembinder et al. (2006), Goldstein, Hotchkiss, and Sirri (2007), and Bessembinder and Maxwell (2008) all find that an increase in transparency leads to a decrease in transaction costs. This is consistent with our model's prediction that the bid–ask spread is lower in a transparent market than in an opaque market. Regarding the impact of transparency on trading activities, Bessembinder and Maxwell ((2008), p. 232) conjecture that "TRACE likely increased traders' willingness to submit electronic limit orders by allowing traders to choose limit prices with enhanced knowledge of market conditions." This is consistent with our result that the probability of trade is higher in transparent markets. However, the empirical evidence on the impact of transparency on trading activity is mixed. Goldstein et al. find from an experiment conducted by the NASD in 2003 that transparency led to no significant change in trading volume of BBB corporate bonds. Asquith, Covert, and Pathak (2013) document a significant decrease in trading activity for high-yield bonds when examining the 2002–2005 expansion of the TRACE coverage. Consistent with our results on trading activity, Bessembinder, Jacobsen, Maxwell, and Venkataraman (2018) in contrast find substantial increases in trading activity among bonds first covered by TRACE in 2003, 2004 and 2014.

## III. Model and Full Information Environment

We assume that the party that initiates the sequence of transactions (the first counterparty) has some private benefit or cost that motivates him to trade; however, he also exploits his private information when trading. Dealers are less informed about the asset value than the first counterparty, and the least informed party is the customer approached by the dealer when the dealer seeks to trade out of the position established in the first transaction. We assume that this second counterparty has the least information about the asset value, because we assume her interest in the asset is solely a function of being contacted by the dealer regarding it. As mentioned in the introduction, we assume this second counterparty is one of many investors who are known to the dealer to invest in assets of the same general type but who have no special information about the particular asset in question. In a transparent market, the second counterparty and the market at large are informed by the price at which the first transaction takes place.

For concreteness, we assume that the first counterparty seeks to sell an asset. The case of a purchase by the first counterparty is symmetric.[3] We call the first counterparty the seller, and we call the second counterparty the buyer. All parties are risk neutral.

We denote the seller's value for the asset as $\tilde{v}-\Delta$. Here, $\tilde{v}$ is a common-value component and is the value to all other market participants (excluding dealers), and $\Delta>0$ is a private-value component that could represent a liquidity shock to the seller (e.g., it could represent the haircut if the asset has to be used as collateral to raise cash). We call $\Delta$ the seller's discount. When trade occurs, $\Delta$ is the gain from trade. The dealer has a signal $\tilde{s}$ about $\tilde{v}$. We call $s$ the dealer's type.

We call the price quoted to the seller the dealer's bid and denote it by $B$, and we call the price quoted to the buyer the dealer's ask and denote it by $A$. The dealer's strategy consists of a bid function $B(\cdot)$ and an ask function $A(\cdot)$ specifying the bid and ask for each dealer type. The dealer quotes an ask price to the buyer only when the dealer's bid has been accepted by the seller. We assume that the dealer has a strong aversion to holding inventories, which we model by taking the dealer's value for the asset to be $-\infty$. In equilibrium, the dealer never ends up holding the asset, and we could replace $-\infty$ by any sufficiently low number. The dealer will choose to participate in the game only if the expected profit is nonnegative, so a requirement for equilibrium (an individual rationality constraint) is that each type of dealer earn nonnegative expected profit.

The seller has a dominant strategy, which is to accept all bids above $\tilde{v}-\Delta$ and to reject all others. We assume that the seller plays his dominant strategy, and we focus on the subgame between the dealer and the buyer. Note that we do not need to assume that the seller knows the dealer's signal $\tilde{s}$, because the seller has a dominant strategy that does not depend on $\tilde{s}$.

*Assumption 1.* The seller's discount $\Delta$ is common knowledge, but the realization of the common value component $\tilde{v}$ is known only to the seller. The value $\tilde{v}$ and the dealer's signal $\tilde{s}$ are linked by $\tilde{v}=\tilde{s}+\tilde{\varepsilon}$, where $\tilde{s}$ and $\tilde{\varepsilon}$ are independently distributed. The support of $\tilde{s}$ is a finite interval $[s_L, s_H]$. The random variable $\tilde{\varepsilon}$ has a continuous density function, its support is the entire real line, and its mean is 0. The distribution function $F$ of $\tilde{\varepsilon}$ is such that $\log F$ is strictly concave, and

$$\lim_{x\to-\infty}\frac{F(x)}{f(x)} = 0. \tag{1}$$

An example that satisfies Assumption 1 is $\tilde{v}=\tilde{s}+\tilde{\varepsilon}$, where $\tilde{s}$ has finite support and $\tilde{\varepsilon}$ is normally distributed with mean zero. We study this example in Section VI. We do not assume normality now, because it provides no simplification

[3]To be more precise, the case of a purchase is symmetric if the dealer can go short in the first transaction. In reality, a dealer may search for a second counterparty in order to buy the asset before executing a sale to the first counterparty, when the first counterparty wishes to buy an asset. In this case, the second counterparty is obviously not informed by the price of the "first" transaction, as we assume in our model.

at this point. Assumption 1 states that the asset value equals the signal plus noise. Some readers may be more familiar with "truth plus noise" signals of the form $\tilde{z}=\tilde{v}+\tilde{e}$, where $\tilde{v}$ and $\tilde{e}$ are independent. Given such a signal $\tilde{z}$, we can always define a new signal $\tilde{s}=\mathsf{E}[\tilde{v}\,|\,\tilde{z}]$ with residual $\tilde{\varepsilon}=\tilde{v}-\tilde{s}$. Then, $\tilde{v}=\tilde{s}+\tilde{\varepsilon}$ and the mean of $\tilde{\varepsilon}$ is zero, as we assume. Furthermore, $\tilde{\varepsilon}$ is mean-independent of $\tilde{s}$, so our assumption that the two are independent is only a mild regularity condition.

Log concavity is a common and important assumption in information economics (see, e.g., Bagnoli and Bergstrom (2005)). The details of its application to our model are provided in the appendices, but we explain the essential idea here. The conditional probability that the seller will accept a bid of $B$, conditioning on the dealer's type $s$, is the probability that $\tilde{v}\equiv\tilde{\varepsilon}+s\le B+\Delta$, which is $F(B+\Delta-s)$. Consider two dealer types $s>s'$. Log concavity implies that the ratio

$$\frac{F(B+\Delta-s)}{F(B+\Delta-s')} \tag{2}$$

is increasing in $B$. Thus, increasing the bid increases the probability of trade at a higher rate for a dealer of a higher type. This makes it possible in the transparent market for a dealer to separate from other dealers with lower types by offering a higher bid. It also makes pooling equilibria implausible in the transparent market, because higher types of dealers have a greater incentive to deviate from pooling behavior by offering higher bids (formally, as shown in Appendix B, pooling equilibria violate the D1 criterion).

In order to see that the ratio (2) is increasing in $B$, define

$$g(x) \quad = \quad \frac{F(x)}{f(x)}. \tag{3}$$

We make extensive use of this function throughout the paper. The log concavity assumption implies that $g$ is an increasing function (this follows from the fact that $(\log F)''=(f/F)'$). The derivative of the log of the ratio (2) is:

$$\frac{1}{g(B+\Delta-s)}-\frac{1}{g(B+\Delta-s')},$$

which is positive for $s>s'$, because $g$ is increasing. Thus, the ratio (2) is increasing in $B$.

It is also useful to define the following function:

$$G(x) \quad = \quad \mathsf{E}[\tilde{\varepsilon}\mid\tilde{\varepsilon}\le x] \quad = \quad \frac{1}{F(x)}\int_{-\infty}^{x}\varepsilon f(\varepsilon)\,\mathrm{d}\varepsilon. \tag{4}$$

The log concavity assumption implies that $G'<1$ (Bagnoli and Bergstrom, ((2005), Lemma 1)). It is also worth noting that the definitions of $g$ and $G$ imply directly that

$$g(x)G'(x) \quad = \quad x-G(x). \tag{5}$$

Of course, $g$ is positive. Condition (1) implies that all positive numbers are in the range of $g$, so $g^{-1}(a)$ is well defined for all $a>0$.

If the buyer knew the dealer's type $s$ and knew that a transaction had occurred between the dealer and the seller at a bid of $B$, then he would know both $s$ and that $\tilde{v} \le B+\Delta$. In this circumstance, the buyer's reservation price for the asset would be:

$$R(s,B) \stackrel{\text{def}}{=} \mathsf{E}[\tilde{v} \mid \tilde{v} \le B+\Delta, \tilde{s}=s] = s+G(B+\Delta-s). \tag{6}$$

In the full information environment, the dealer of type $s$ chooses her bid $B$ to maximize

$$[R(s,B)-B]\,\text{prob}(\tilde{v} \le B+\Delta \mid \tilde{s}=s) = [R(s,B)-B]F(B+\Delta-s). \tag{7}$$

All proofs are in Appendix A.

*Theorem 1.* Set $\gamma = g^{-1}(\Delta)$. In the full information environment, there is a unique equilibrium. In equilibrium, the dealer of type $s$ bids

$$B^{\dagger}(s) = s+\gamma-\Delta, \tag{8}$$

and plays the ask

$$A^{\dagger}(s) = R(s, B^{\dagger}(s)) = s+G(\gamma). \tag{9}$$

The conditional probability of trade, the bid–ask spread, and the dealer's conditional expected profit are all independent of the dealer's type. For each type of dealer, the conditional probability of trade is $F(\gamma)$, the bid–ask spread is:

$$\Delta+G(\gamma)-\gamma = [1-G'(\gamma)]g(\gamma) > 0, \tag{10}$$

and the conditional expected profit is $[1-G'(\gamma)]g(\gamma)F(\gamma)>0$.

## IV. The Transparent Market

In a transparent market, the buyer observes the bid and ask and forms beliefs about $\tilde{v}$ and $\tilde{s}$. We search for an equilibrium in which different dealer types make different bids to the seller—that is, we search for an equilibrium that is separating in bids. The reason we do so is that an equilibrium with pooling in bids violates the D1 criterion. The details are provided in Appendix B. Intuitively, higher dealer types have more incentive to deviate from a pooling equilibrium by playing a higher bid, because increasing the bid increases the probability of trade more for a dealer with a higher signal (the ratio (2) is increasing in $B$ when $s>s'$). Furthermore, pooling in bids implies pooling in asks, because if two different dealer types play the same bid $B$, then the highest ask that is accepted by the buyer after observing an initial transaction at $B$ is optimal for both dealer types. Therefore, the dealer type for which the probability of trade increases more is the type that has the most to gain from deviating to a higher bid. Hence, the buyer should (under the D1 criterion) infer that a deviation to a higher bid is made by a dealer with a higher type, making that deviation profitable for the higher type. Thus, the D1 criterion rules out equilibria that are pooling in bids. The actual argument is only slightly more complicated than this: It relies on the fact that the marginal rate of substitution (B-1) is increasing in the dealer's type, which is a consequence of log concavity.

When contacted by the dealer, the buyer knows that the dealer has acquired inventory in a prior transaction, so the buyer's beliefs place a probability of 1 on the dealer's bid having been accepted by the seller, which is the event $\tilde{v} \leq B(\tilde{s}) + \Delta$. In an equilibrium that is separating in bids, the buyer therefore knows the realization $s$ of $\tilde{s}$ and he knows that $\tilde{v}$ satisfies $\tilde{v} \leq B(s) + \Delta$ when he is contacted by the dealer, before the dealer quotes an ask. The highest ask price the buyer would pay given that information is the reservation price $R(s, B(s))$ defined in equation (6). Hence, we look for an equilibrium that is separating in bids and in which the ask price is $R(s, B(s))$. We discuss whether there could be other separating equilibria—in which the ask price is different from $R(s, B(s))$—at the end of this section. To derive the equilibrium, we first need a technical lemma.

*Lemma 1.* Set $x_L = g^{-1}(\Delta)$. There exists $x_H < \infty$ such that:

$$s_L + \int_{x_L}^{x_H} \frac{g(a) - \Delta}{G(a) + \Delta - a} \, \mathrm{d}a \quad = \quad s_H. \tag{11}$$

For $x_L \leq x \leq x_H$, define:

$$s(x) \quad = \quad s_L + \int_{x_L}^{x} \frac{g(a) - \Delta}{G(a) + \Delta - a} \, \mathrm{d}a. \tag{12}$$

The function $s(\cdot)$ is strictly increasing on the interval $[x_L, x_H]$ with range equal to $[s_L, s_H]$.

The next theorem describes the equilibrium. Formula (12) in Lemma 1 is important, because it implies that the inverse function appearing in Theorem 2 solves an ODE that is equivalent to the dealer's first-order condition. We demonstrate this in the proof of Theorem 2, and we also show that, due to log concavity, the first-order condition is sufficient for the optimum. We obtained formula (12) by integrating the inverse ODE. These steps—deriving an ODE from the first-order condition and then analyzing the inverse ODE—follow Mailath (1987).

*Theorem 2.* Let $x(\cdot)$ be the inverse of function (12) with domain $[s_L, s_H]$. Then, $x(s_L) = x_L \equiv g^{-1}(\Delta)$. For each dealer type $s$, define:

$$B^*(s) \quad = \quad s + x(s) - \Delta. \tag{13}$$

Consider the following beliefs for the buyer:

- If the dealer's bid $B$ is equal to $B^*(s)$ for some $s \in [s_L, s_H]$, then, for any ask $A$, the buyer places a probability of 1 on $\tilde{s} = s$.

- If the dealer's bid $B$ is less than $B^*(s_L)$ then, for any ask $A$, the buyer places a probability of 1 on $\tilde{s} = s_L$.

- If the dealer's bid $B$ is greater than $B^*(s_H)$ then, for any ask $A$, the buyer places a probability of 1 on $\tilde{s} = s_H$.

In addition, consider the following strategy for the buyer: The buyer accepts the dealer's ask $A$ if and only if either

- the dealer's bid $B$ is equal to $B^*(s)$ for some $s \in [s_L, s_H]$ and $A \le R(s,B)$, or
- the dealer's bid $B$ is less than $B^*(s_L)$ and $A \le R(s_L, B)$, or
- the dealer's bid $B$ is greater than $B^*(s_H)$ and $A \le R(s_H, B)$.

The bid function $B^*$ and ask function

$$A^*(s) \quad = \quad R(s, B^*(s)) \quad = \quad s + G(x(s)) \tag{14}$$

constitute a separating Bayesian-Nash equilibrium in conjunction with the buyer's beliefs and strategy just described. In this equilibrium, the conditional probability of trade is $F(x(s))$, which is strictly increasing in $s$. The bid–ask spread is $\Delta + G(x(s)) - x(s)$, which is strictly decreasing in $s$, and the dealer's conditional expected profit is:

$$[1 - G'(x_L)]g(x_L)F(x_L)\exp\left(-\int_{s_L}^{s} \frac{1}{g(x(a))}\,\mathrm{d}a\right), \tag{15}$$

which is positive and strictly decreasing in $s$.

Given the strategy of the buyer specified in the theorem, the expected profit that can be achieved by a dealer of type $s$ who plays the bid $B$ and is believed to be of type $s'$ is:

$$\begin{aligned} U(s, s', B) &\overset{\text{def}}{=} [R(s', B) - B]F(B + \Delta - s) \\ &= [s' + G(B + \Delta - s') - B]F(B + \Delta - s). \end{aligned} \tag{16}$$

In equilibrium, the dealer of type $s$ plays the bid $B^*(s)$ and is correctly perceived to be of type $s$, thus she earns an expected profit equal to $U(s, s, B^*(s))$. As is standard, the worst type of dealer plays the same strategy and earns the same expected profit as in the full information environment; that is, $B^*(s_L) = B^\dagger(s_L)$. The incentive compatibility condition for a separating equilibrium with bid function $B(\cdot)$ is that $s' = s$ solves $\max_{s'} U(s, s', B(s'))$ for each $s$. The first-order condition for this maximization problem produces an ODE. As stated previously, function (12) solves the inverse of this ODE subject to the boundary condition $B^*(s_L) = B^\dagger(s_L)$.

If we substitute $x_L = g^{-1}(\Delta)$ for $x(s)$ in the bid function (13) and ask function (14) of the transparent market, then we recover the bid function and ask function in the full information environment. Because $x(s) > x_L$ for $s \neq s_L$, we see that the desire to separate from lower types induces dealers to bid higher in the transparent market. We also see that dealers quote higher asks in the transparent market. The higher asks are possible because the average quality of assets purchased by the dealer is higher, due to the higher bids. The bid–ask spread is lower in the transparent market than in the full information environment; that is,

$$\Delta + G(x(s)) - x(s) \quad < \quad \Delta + G(x_L) - x_L,$$

for $s \neq s_L$, because $G' < 1$. Furthermore, the expected profit is lower in the transparent market. This is because, as formula (15) shows, the expected profit is less than $[1 - G'(x_L)]g(x_L)F(x_L)$, which is the expected profit of each dealer type in the full information environment. Of course, the dealer's profit cannot be higher in a separating equilibrium of the transparent market than when the dealer's signal is publicly observable. This is the standard result that signaling is costly, relative to the full information environment.

*On the Uniqueness of the Separating Equilibrium.* The issue we address here is whether there exist separating equilibria other than that in Theorem 2. We show in Appendix B that the equilibrium in Theorem 2 is the unique equilibrium that is separating in bids in which the bid function $B(\cdot)$ is differentiable and the ask price is the buyer's reservation price $R(s, B(s))$. Thus, the main question is whether the ask price in a separating equilibrium can be some function $\hat{A}(s) \neq R(s, B(s))$. Given a separating bid function $B(\cdot)$, the ask price cannot be higher than the buyer's reservation price, because higher prices are rejected by the buyer. Thus, the question is whether it is possible that $\hat{A}(s) < R(s, B(s))$. Such an ask price would give rents to the buyer, so an equivalent question is whether it is possible for the buyer to earn rents in a separating equilibrium.

There is an alternate version of our model in which the question is easy to answer. Suppose that, instead of quoting an ask price, the dealer auctions the asset to buyers. Because the buyers are uninformed, the auction would be a Bertrand game, and competition between the buyers would eliminate their rents and result in a transaction price $R(s, B(s))$. Thus, in the auction version of the model, there is a unique separating equilibrium in which the bid function is differentiable, and the equilibrium is the one displayed in Theorem 2.

In the model in which the dealer quotes an ask price to the buyer, the question is more complex, because the party taking the action (the dealer) is the informed party. Thus, the ask price can signal to the buyer, even when dealer types have already been separated by the bid. This issue is similar to the issue that arises when a seller of a good can signal via both advertising and price. Kihlstrom and Riordan (1984) assume that sellers of different quality goods that separate via advertising can price the good at the customer's reservation price, given the customer's awareness of quality achieved via advertising. In our context, this approach implies that the ask is set at the buyer's reservation price, after the dealers have separated via their bids, that is, the ask price is $R(s, B(s))$, as in Theorem 2. On the other hand, Milgrom and Roberts (1986) assume that both advertising and price signal. In this "dual signals" model, in our context, there are multiple separating equilibria. The reason is as follows. Even though $R(s, B(s))$ would be the buyer's reservation price given knowledge that $\tilde{s} = s$ and $\tilde{v} \leq B(s) + \Delta$, it is possible that the buyer expects the dealer of type $s$ to quote a different ask $\hat{A}(s) < R(s, B(s))$. Furthermore, it is possible that the buyer would, if quoted $R(s, B(s))$, make the (out-of-equilibrium) inference that the dealer's type is $s' < s$ and hence would reject the ask $R(s, B(s))$. In this circumstance, it may be optimal for the dealer to quote $\hat{A}(s)$, producing an equilibrium with ask prices below $R(s, B(s))$. Equilibria of this sort do not seem to be ruled out by standard refinements, because all dealers have the same preferences regarding ask prices: their utility goes up one-for-one with the ask price up to the point that the ask is rejected by the buyer.

Despite the existence of the equilibria described in the preceding paragraph, we feel that the equilibrium in Theorem 2 is the natural focal point. First, it is optimal for the dealer, who is the party taking the actions (quoting the bid and ask) because it gives no rents to the buyer. Second, the other equilibria require a nonmonotonicity of beliefs that seems unreasonable. In the scenario described in the preceding paragraph, the buyer thinks the dealer's type is $s$ when he observes an ask of $\hat{A}(s)$, but he infers that the dealer's type is $s' < s$ when he observes an ask of $R(s, B(s)) > \hat{A}(s)$. If we suppose more reasonably that either inferences are unchanged by asks or that inferences are monotone in asks, then the ask price must be $R(s, B(s))$ in any equilibrium that is separating in bids. We demonstrate this formally in Appendix B. We show that the equilibrium in Theorem 2 is the unique separating equilibrium in which the bid function is differentiable and in which the buyer's inferences are weakly monotone in the ask price.

## V. The Opaque Market

In an opaque market, the buyer does not observe the price of the transaction between the seller and the dealer. The buyer's only information is that the transaction occurred. Let $B^o(\cdot)$ denote an equilibrium bid function in the opaque market. The buyer knows that $\tilde{v} \le B^o(\tilde{s}) + \Delta$, but he knows neither $\tilde{v}$ nor $\tilde{s}$. The buyer's reservation price given this information is:

$$\bar{R} \quad \stackrel{\text{def}}{=} \quad \mathsf{E}[\tilde{v} \mid \tilde{v} \le B^o(\tilde{s}) + \Delta]. \tag{17}$$

All dealer types must play the same ask in equilibrium, because all dealer types will play the highest ask the buyer will accept. Given a planned ask price of $A$, the dealer of type $s$ chooses a bid $B$ to maximize expected profit:

$$(A - B) \times \text{prob}(\tilde{v} \le B + \Delta \mid \tilde{s} = s) \quad = \quad (A - B) \times F(B + \Delta - s). \tag{18}$$

For all real $x$, define

$$h(x) \quad = \quad x + g(x),$$

where $g(x) = F(x)/f(x)$ as before. In the proof of the next theorem, we show that, given a planned ask price of $A$, the optimal bid is

$$B(s) \quad = \quad s - \Delta + h^{-1}(A + \Delta - s). \tag{19}$$

Substituting this into the reservation price (17), we see that the buyer's reservation price in equilibrium is

$$\begin{aligned} \bar{R} \quad &= \quad \mathsf{E}[\tilde{v} \mid \tilde{v} \le \tilde{s} + h^{-1}(A + \Delta - \tilde{s})] \\ &= \quad \mathsf{E}[\tilde{s} + \tilde{\varepsilon} \mid \tilde{\varepsilon} \le h^{-1}(A + \Delta - \tilde{s})] \\ &= \quad \mathsf{E}[\tilde{s} + \tilde{\varepsilon} \mid \tilde{s} + h(\tilde{\varepsilon}) \le A + \Delta]. \end{aligned} \tag{20}$$

It seems reasonable that the dealer should set the ask at the buyer's reservation price. However, there is a uniqueness issue similar to that in the transparent market. Equilibria in which the ask is less than the buyer's reservation price, so the buyer earns rents, can be sustained by off-equilibrium beliefs. Also as in the transparent market, such off-equilibrium beliefs cannot be monotone in the ask (higher asks must be viewed as coming from dealers with lower types). We show in Appendix B that the equilibrium ask must be equal to the buyer's reservation price in any equilibrium in which the buyer's beliefs are weakly monotone in the ask price. An equilibrium of this sort is computed by setting $A=\bar{R}$ and solving equation (20) for $A$. We verify numerically in the next section that this equation has a solution (see Figure 1 in particular).

*Theorem 3.* Suppose $A=A^o$ is a solution of the equation

$$A \quad = \quad \mathsf{E}[\tilde{s}+\tilde{\varepsilon} \mid \tilde{s}+h(\tilde{\varepsilon}) \leq A+\Delta]. \tag{21}$$

Set $y(s)=h^{-1}(A^o+\Delta-s)$. It is an equilibrium of the opaque market for all dealer types to quote the ask price $A^o$ and for the dealer of type $s$ to bid:

$$B^o(s) \quad = \quad s+y(s)-\Delta. \tag{22}$$

In this equilibrium, the conditional probability of trade is $F(y(s))$, which is strictly decreasing in $s$. The bid–ask spread is $g(y(s))$, which is strictly decreasing in $s$. The dealer's conditional expected profit is $g(y(s))F(y(s))$, which is positive and strictly decreasing in $s$.

One difference between transparent and opaque markets is that the probability of trade is higher for higher values of $s$ in a transparent market (because the equilibrium bid rises more than one-for-one with the dealer's signal) but falls with $s$ in an opaque market (because the equilibrium bid rises less than one-for-one with the dealer's signal). Expected profit falls with $s$ in both markets, but it falls faster in an opaque market, because both the probability of trade and the bid–ask spread are lower for higher $s$ in an opaque market. This produces the very intuitive result that higher dealer types may ex post prefer transparency, because transparency allows them to signal their types. In Section VI, we look at ex ante dealer profits in the two markets.

## VI. A Uniform/Normal Model

To assess the effects of transparency, relative to an opaque market, we need to calculate the integral in (12) and numerically compute the fixed point in equation (21). Thus, we now impose some additional structure on the model. We assume the dealer's signal is uniformly distributed on an interval $[s_L, s_H]$, and we assume $\tilde{v}=\tilde{s}+\tilde{\varepsilon}$, where $\tilde{\varepsilon}$ is normally distributed with mean zero and variance $\sigma^2$ (the normal distribution has all of the properties we have assumed for $\tilde{\varepsilon}$). From numerical analysis of this model, we reach the following conclusions: Transparency increases both bid and ask prices and therefore increases volume, but it reduces bid–ask spreads. The welfare of the seller is improved by the higher bid prices, and the buyer does not earn any rents in either market by assumption, thus the aggregate welfare of investors is higher under transparency. Whether dealers gain or lose from transparency depends on whether the lower spreads or the higher volume is more important. We find that the lower spreads are more important if adverse selection is low or if the seller's discount is high. In either situation, volume is high even in an opaque market, so the reduction of spreads is the more important phenomenon, and dealer profits fall when transparency is introduced.

In presenting the numerical results, we express the bid and ask prices and the seller's discount $\Delta$ as percentages of the unconditional expected asset value, which is $(s_L+s_H)/2$. We also parameterize the adverse selection in terms of the total amount of private information (seller's plus dealer's) and the fraction of the total that is explained by the dealer's information. The variance of the uniformly distributed $\tilde{s}$ is $(s_H-s_L)^2/12$, so the variance of $\tilde{v}=\tilde{s}+\tilde{\varepsilon}$ (the total amount of private information) is:

$$\phi^2 \overset{\text{def}}{=} \frac{(s_H-s_L)^2}{12}+\sigma^2. \tag{23}$$

The fraction of the variance of $\tilde{v}$ that is explained by $\tilde{s}$ is $\rho^2$, where $\rho$ is the correlation of $\tilde{s}$ and $\tilde{v}$; that is,

$$\rho \overset{\text{def}}{=} \frac{s_H-s_L}{\phi\sqrt{12}}. \tag{24}$$

We can exogenously specify the total amount of private information $\phi^2$ and the fraction $\rho^2$ that is explained by the dealer's signal and then recover $s_H-s_L$ and $\sigma$ as $s_H-s_L=\rho\phi\sqrt{12}$ and $\sigma^2=(1-\rho^2)\phi^2$, respectively.

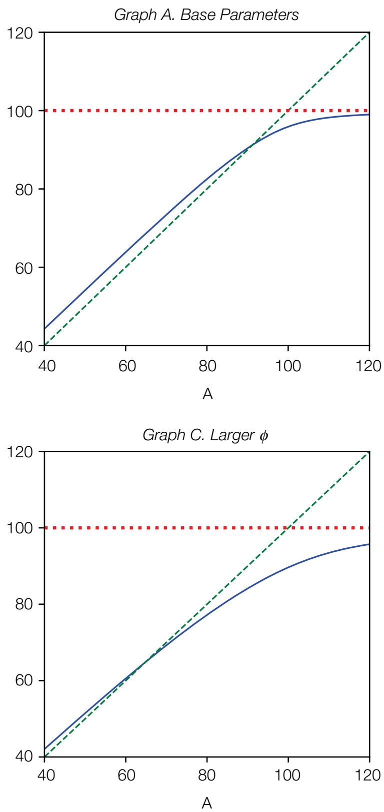
Sections III–V present analytic solutions for all quantities of interest other than the fixed point in equation (21) in the opaque market. Figure 1 illustrates the calculation of the fixed point. The figure demonstrates that the equilibrium $A^o$ equals the unconditional expected asset value minus a discount for adverse selection. The discount for adverse selection is smaller when the seller's discount $\Delta$ is larger and when the total amount of private information $\phi$ is smaller.

Figure 2 illustrates the equilibria in the three markets: Transparent, opaque, and full information. The figure illustrates the general properties established in the theorems. The bid price is increasing in the dealer's signal in all three markets. The ask price is increasing in the signal in the transparent market and in the full information environment, but it is independent of the signal in the opaque market. In the full information environment, the bid–ask spread, the conditional probability of trade, and the conditional expected profit are all independent of the dealer's signal. In both the transparent and the opaque market, the bid–ask spread and the dealer's expected profit are decreasing in her signal. However, the conditional probability of trade is increasing in the dealer's signal in the transparent market and decreasing in the dealer's signal in the opaque market.

Figure 2 illustrates the relationship between the transparent market and the full information environment that is discussed in Section IV. Bid prices are higher in the transparent market, because the dealer bids higher in order to separate from dealers with lower signals. The ask price is also higher but does not rise as much as the bid, thus the bid–ask spread is lower in the transparent market. Due to the higher bid price, the probability of trade is higher in the transparent market, but, from the dealer's point of view, it does not rise enough to offset the lower bid–ask spread. Hence, the dealer's expected profit is lower in the transparent market.

FIGURE 1
**Equilibrium Ask in the Opaque Market**

The solid curves in Figure 1 depict the function of $A$ that is on the right-hand side of (21), with $A$ expressed as a percentage of the mean asset value $(s_L + s_H)/2$. The equilibrium ask is at the intersection with the 45° line. In Graph A, the standard deviation of the asset value is 5% of the mean asset value, the seller's discount is 5% of the mean asset value, and 25% of the variance of $\tilde{v}$ is explained by the dealer's signal. The parameters in Graph B are the same, except that the seller's discount is 10% of the mean asset value. The parameters in Graph C are the same as in Graph A, except that the standard deviation of the asset value is 10% of its mean. The parameters in Graph D are the same as in Graph A, except that 75% of the variance of the asset value is explained by the dealer's signal.

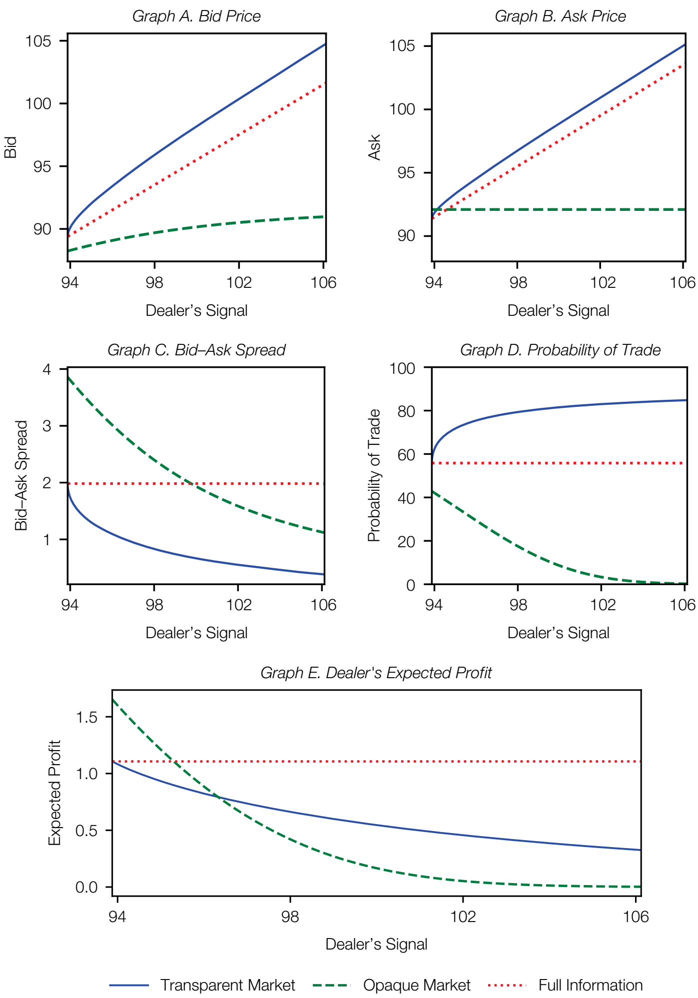



All of the properties of Figure 2 discussed to this point were established analytically in earlier sections. Figure 2 provides new insights on the relationship between the transparent and opaque markets, the central issue with which this paper is concerned. For the particular parameter values used in Figure 2, we see that the bid price in the opaque market is uniformly lower than in the transparent market. Hence, the probability of trade is uniformly lower in the opaque market. This means that the aggregate realized gains from trade are lower in the opaque market. Furthermore, the bid–ask spread is uniformly higher in the opaque market than in the transparent market. Whether the dealer gains or loses from transparency depends on whether the beneficial effect of transparency on the spread offsets the detrimental effect of transparency on volume. We see from Graph E that the former effect dominates when the dealer's signal is low, and the latter effect dominates when the dealer's signal is high. This is intuitive. In an opaque

FIGURE 2

Equilibria in the Three Markets

Figure 2 presents the separating equilibrium of the transparent market, the equilibrium of the opaque market, and the equilibrium in the full information environment. All variables other than the probability of trade (which is in percentage) are expressed as a percentage of the mean asset value $(s_L+s_H)/2$. In this example, the seller's discount $\Delta$ is 5% of the mean asset value, the standard deviation $\phi$ of the asset value is 5% of the mean asset value, and 50% of the variance of the asset value is explained by the dealer's signal.




market, the ask price is independent of the dealer's signal. When the dealer has a low signal, she can expect the seller to accept a low bid, producing a relatively high likelihood of trade with a high spread. Hence, opacity is desirable for the dealer conditional on the signal being low. Conversely, when the dealer has a high signal, she knows she has to bid high to buy the asset, producing a low spread. This diminishes the incentive to bid high, resulting in both a low probability of trade and a low spread, both of which contribute to a low expected profit. Thus, ex post, the dealer prefers opacity when her signal is low and prefers transparency when her signal is high. We examine the dealer's ex ante preferences regarding transparency in Figures 3 and 4.

Figure 3 shows that the effect of transparency on the bid–ask spread and the probability of trade that is indicated in Figure 2 is robust when we vary one parameter at a time. For various values of the seller's discount, the total amount of private information, and the fraction of private information that is explained by the dealer's signal, we see in Graphs A, D, and G that transparency increases the probability of trade, and we see in Graphs B, E, and H that transparency reduces the

FIGURE 3

Comparative Statics

Figure 3 compares the separating equilibrium of the transparent market and the equilibrium of the opaque market for various parameter values. With the exception of the probability of trade and the fraction $\rho^2$ of the variance of $\tilde{v}$ that is explained by the dealer's signal (which are in percentages), all variables are expressed as a percentage of the mean asset value $(s_L + s_H)/2$. The base parameters are that the seller's discount $\Delta$ is 5% of the mean asset value, the standard deviation $\phi$ of $\tilde{v}$ is 5% of the mean asset value, and 25% of the variance of $\tilde{v}$ is explained by the dealer's signal. Each row shows the effects of varying a single parameter, relative to the base case.

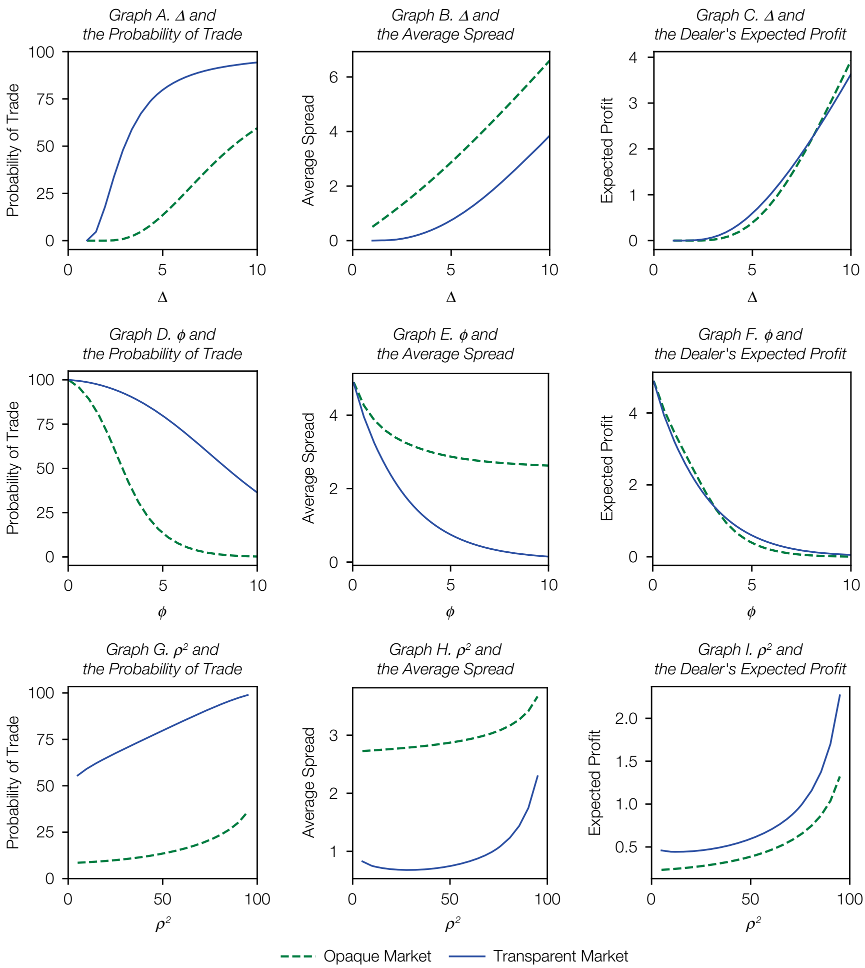
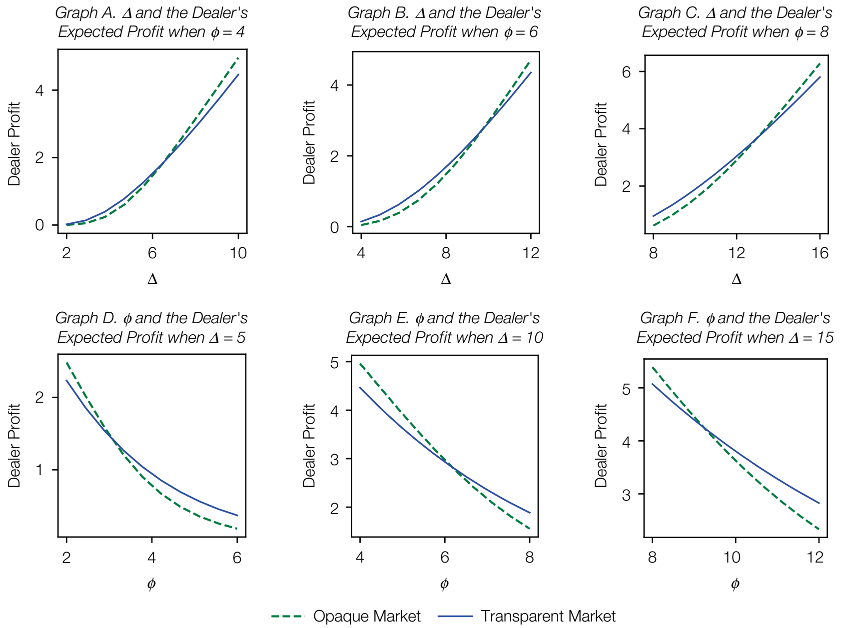


FIGURE 4
Comparative Statics of Dealer Profits

Figure 4 shows expected dealer profits in the separating equilibrium of the transparent market and in the equilibrium of the opaque market. All variables are expressed as a percentage of the mean asset value $(s_L + s_H)/2$. In each case, 50% of the variance of $\tilde{v}$ is explained by the dealer's signal. The standard deviation of $\tilde{v}$ (denoted by $\phi$) and the seller's discount (denoted by $\Delta$) are as shown.




bid–ask spread. Both effects cause investors to benefit from transparency. Whether dealers prefer transparency or opacity depends on which of the two effects dominate. When potential gains from trade are large, or when the dealer has little private information, trade is likely in either an opaque or transparent market (see Graphs A and D of Figure 3). In this circumstance, the beneficial effect of transparency on volume is relatively small and is offset by the detrimental effect on the bid–ask spread from the point of view of dealers. This is shown in Graphs C and F of Figure 3. Graph C in Figure 3 shows that transparency is beneficial to dealers when the potential gains from trade are small and detrimental when potential gains from trade are large. Graph F shows that transparency is beneficial to dealers when the dealer has a lot of private information and detrimental when private information is small. Graphs G–I in Figure 3 show that the fraction of the variance of $\tilde{v}$ that is explained by the dealer's signal seems to have little effect on the relative performance of the two markets. We study the remaining two parameters ($\Delta$ and $\phi$) further in Figure 4.

Figure 4 provides more insight into the effect of transparency on dealer profits by varying $\Delta$ and $\phi$ simultaneously. For all values of $\phi$, the dealer prefers transparency when the seller's discount $\Delta$ is small and prefers opacity when the seller's discount is large. Conversely, for all values of $\Delta$, the dealer prefers opacity when $\phi$ is small and prefers transparency when $\phi$ is large. These results are consistent with the intuition described earlier: When either adverse selection is low or the seller's private motive for trade is large, then trade is very likely in either market. Thus, the lower spreads induced by transparency are more important than the increase in volume due to transparency, and dealer profits fall when transparency is introduced.

## VII. Conclusion

When markets are ex post transparent and dealers have information not possessed by the second counterparty in a round-trip transaction, they engage in costly signaling, offering unduly favorable prices in the first transaction to signal to the second counterparty. This costly signaling increases volume, liquidity, and market efficiency. From the perspective of dealers, the lower spreads and higher volume are partially offsetting factors. Which is more important depends on the magnitude of potential gains from trade and the extent of adverse selection. Dealers prefer opacity when potential gains from trade are high relative to adverse selection. Given the frequent opposition of dealers to transparency, we conclude that this condition characterizes many OTC markets.

## Appendix A. Proofs

Recall the definitions of $g$ and $G$ in (3) and (4). As pointed out in (5), the definitions imply directly that $g(x)G'(x) = x - G(x)$. Recall also that the strict log concavity of $F$ implies that $g' > 0$ and $G' < 1$.

*Proof of Theorem 1.* The dealer's expected profit (7) is equal to

$$[s + G(B + \Delta - s) - B]F(B + \Delta - s).$$

For simplicity, set $x = B + \Delta - s$, so we can write the expected profit as:

$$[G(x) + \Delta - x]F(x).$$

Maximizing in $x$ is equivalent to maximizing in $B$. The claim in (8) is that the optimal $x$ is $g^{-1}(\Delta)$. The derivative of expected profit with respect to $x$ is:

$$[G'(x) - 1]F(x) + [G(x) + \Delta - x]f(x).$$

To prove the claim about the optimum, it suffices to show that this derivative is positive for $x < g^{-1}(\Delta)$ and negative for $x > g^{-1}(\Delta)$. Making the substitution $g(x)G'(x) = x - G(x)$, we see that the derivative is equal to:

$$[G'(x) - 1]g(x)f(x) + [G(x) + \Delta - x]f(x) \quad = \quad [\Delta - g(x)]f(x).$$

By the monotonicity of $g$, $\Delta - g(x)$ is positive for $x < g^{-1}(\Delta)$ and negative for $x > g^{-1}(\Delta)$. This completes the proof that (8) is the unique optimal bid.

The probability of trade for a dealer of type $s$ is $F(B^{\dagger}(s) + \Delta - s)$. From (8), the probability of trade is $F(g^{-1}(\Delta))$, for all $s$. The equilibrium ask price for a dealer of type $s$ is

$$R(s, B^{\dagger}(s)) \quad = \quad s + G(B^{\dagger}(s) + \Delta - s) \quad = \quad s + G(g^{-1}(\Delta)).$$

Therefore, the equilibrium bid–ask spread for a dealer of type $s$ is:

$$R(s, B^{\dagger}(s)) - B^{\dagger}(s) \quad = \quad G(g^{-1}(\Delta)) + \Delta - g^{-1}(\Delta).$$

Substitute $x-G(x)=g(x)G'(x)$ for $x=\gamma=g^{-1}(\Delta)$ to obtain the formula in the theorem. The bid–ask spread is positive, because $G'<1$. The expected profit is the bid–ask spread multiplied by the probability of trade. □

*Proof of Lemma 1.* We first need to show that there exists $x_H$ satisfying (11). Note that because $G'(x)=[x-G(x)]/g(x)$ and $G'<1$ (due to log concavity), we have $G(x_L)+g(x_L)>x_L$. Furthermore, by definition, $g(x_L)=\Delta$. Hence, $G(x)+\Delta-x>0$ at $x=x_L$. Define

$$x_H^* \quad = \quad \inf\{a > x_L \mid G(a)+\Delta-a \le 0\}.$$

The function $G(a)+\Delta-a$ is monotone decreasing ($G'<1$); hence, it is positive below $x_H^*$ and negative above. Furthermore, $g(a)-\Delta>0$ for $a>x_L$ due to the definition $x_L=g^{-1}(\Delta)$ and the monotonicity of $g$. Thus, the integrand in (11) is positive for $x$ between $x_L$ and $x_H^*$. Our goal is to show that

$$\int_{x_L}^{x_H^*} \frac{g(a)-\Delta}{G(a)+\Delta-a}\,\mathrm{d}a \quad = \quad \infty.$$

Therefore, there exists some $x_H \in (x_L, x_H^*)$ such that (11) holds.

From the definition of $x_H^*$, $\Delta=x_H^*-G(x_H^*)$. By substituting this term and $g(x_L)=\Delta$, and changing the variables to $z=x_H^*-a$, we obtain:

$$\text{(A-1)} \qquad \int_{x_L}^{x_H^*} \frac{g(a)-\Delta}{G(a)+\Delta-a}\,\mathrm{d}a \quad = \quad \int_0^{x_H^*-x_L} \frac{g(x_H^*-z)-g(x_L)}{z+G(x_H^*-z)-G(x_H^*)}\,\mathrm{d}z.$$

Consider any $x$ between 0 and $x_H^*-x_L$. The integral (A-1) is at least as large as the integral of the same integrand from 0 to $x$. Between 0 and $x$, the numerator of the integrand is at least as large as $g(x_H^*-x)-g(x_L)>0$. Also, $G(x_H^*-z)-G(x_H^*)<0$, so the denominator, while positive, is smaller than $z$. Therefore, the integral is at least as large as

$$[g(x_H^*-x)-g(x_L)]\int_0^x \frac{1}{z}\,\mathrm{d}z \quad = \quad \infty.$$

This completes the proof that there exists some $x_H \in (x_L, x_H^*)$ such that (11) holds. We have already observed that the integrand in (12) is positive for $a$ between $x_L$ and $x_H^*$. This implies that $s$ defined in (12) is strictly monotone. □

*Proof of Theorem 2.* Clearly, the buyer's strategy is optimal given the beliefs, and the beliefs are consistent with Bayes' rule. We need to show that the dealer's strategy is optimal.

The dealer's strategy that we claim is an equilibrium strategy is: $B^*(s)=x(s)-\Delta+s$, where $x(\cdot)$ is the inverse of (12). For convenience, we drop the $^*$ on $B^*$. From the condition $x(s_L)=x_L=g^{-1}(\Delta)$, we have $B(s_L)=g^{-1}(\Delta)-\Delta+s_L=B^\dagger(s_L)$. Both here and in the full information environment, the ask prices are $R(s,B(s))$. Hence, Theorem 1 shows that:

$$\text{(A-2)} \qquad U(s_L,s_L,B(s_L)) \quad = \quad [1-G'(x_L)]g(x_L)F(x_L) \quad > \quad 0.$$

Consider a dealer of type $s$. To show that $B(s)$ is the optimal bid, given the buyer's strategy, we need to show that:

$$\text{(A-3)} \qquad (\forall s') \qquad U(s,s,B(s)) \quad \ge \quad U(s,s',B(s')),$$

$$\text{(A-4)} \qquad (\forall B < B(s_L)) \qquad U(s,s,B(s)) \quad \ge \quad U(s,s_L,B),$$

$$\text{(A-5)} \qquad (\forall B > B(s_H)) \qquad U(s,s,B(s)) \quad \ge \quad U(s,s_H,B).$$

Condition (A-3) states that $s'=s$ solves the maximization problem $\max_{s'} U(s,s',B(s'))$. Using subscripts to denote partial derivatives, we have:

$$\text{(A-6)} \qquad \frac{\mathrm{d}}{\mathrm{d}s'}U(s,s',B(s')) \quad = \quad U_2(s,s',B(s'))+U_3(s,s',B(s'))\frac{\mathrm{d}B(s')}{\mathrm{d}s'}.$$

The first-order condition for $s'=s$ to be maximal is that

$$U_2(s,s,B(s)) + U_3(s,s,B(s))\frac{\mathrm{d}B(s)}{\mathrm{d}s} \quad = \quad 0. \tag{A-7}$$

The first-order condition is sufficient for optimality (that is, for (A-3)) if the derivative (A-6) is nonnegative for $s'<s$ and nonpositive for $s'>s$.

From (16), we have

$$\begin{aligned} U_2(s,s',B) &= [1-G'(B+\Delta-s')]\times F(B+\Delta-s), \\ U_3(s,s',B) &= [G'(B+\Delta-s')-1]\times F(B+\Delta-s) \\ &\quad +[s'+G(B+\Delta-s')-B]\times f(B+\Delta-s). \end{aligned}$$

After substituting these, as well as $x(s)=B(s)+\Delta-s$, dividing by $f(x(s))$, and substituting $g=F/f$ and $g'G=x-G$, the first-order condition (A-7) becomes:

$$G(x(s))+g(x(s))-x(s)+[\Delta-g(x(s))]\frac{\mathrm{d}B(s)}{\mathrm{d}s} \quad = \quad 0. \tag{A-8}$$

This holds for all $s\in(s_L,s_H)$, because

$$B'(s) \quad = \quad x'(s)+1 \quad = \quad \frac{1}{s'(x(s))}+1 \quad = \quad \frac{G(x(s))+g(x(s))-x(s)}{g(x(s))-\Delta}.$$

Note also that $U_2>0$ because $G'<1$. Therefore, by Theorem 6 of Mailath and von Thadden (2013), the derivative (A-6) is nonnegative for $s'<s$ and nonpositive for $s'>s$ if

$$\frac{U_3(s,s',B(s'))}{U_2(s,s',B(s'))} \tag{A-9}$$

is an increasing function of $s$ for each $s'$. Making the same substitutions we used to derive (A-8), we can calculate the ratio (A-9) as:

$$-1+\frac{G(x(s'))+\Delta-x(s')}{[1-G'(x(s'))]g(B(s')+\Delta-s)}. \tag{A-10}$$

We need to show that the ratio in (A-10) is increasing in $s$. The denominator is positive and decreasing in $s$, because $G'<1$ and $g$ is strictly increasing. Note that the formula (16) for $U$ gives us:

$$U(s',s',B(s')) \quad = \quad [G(x(s'))+\Delta-x(s')]F(x(s')).$$

Thus the sign of the numerator in (A-10) is the same as the sign of $U(s',s',B(s'))$. The formula (15) shows that $U(s,s,B(s))>0$ for all $s$, so we can complete the proof of (A-3) by verifying (15).

The first-order condition (A-7) implies that:

$$\frac{\mathrm{d}}{\mathrm{d}s}U(s,s,B(s)) \quad = \quad U_1(s,s,B(s)).$$

Furthermore,

$$U_1(s,s,B(s)) \quad = \quad -[G(x(s))+\Delta-x(s)]f(x(s)) \quad = \quad -\frac{U(s,s,B(s))}{g(x(s))}.$$

Hence, $\mathrm{d}U/U=-1/g$, which implies that:

$$U(s,s,B(s)) \quad = \quad U(s_L,s_L,B_L)\exp\left(-\int_{s_L}^{s}\frac{1}{g(x(a))}\,\mathrm{d}a\right).$$

Substituting (A-2) yields (15).

Now, we have completed the proof of (A-3). We still need to establish (A-4) and (A-5). First, consider any bid $B < B(s_L)$. The dealer's expected profit at that bid is:

$$U(s, s_L, B) = U(s_L, s_L, B) \times \frac{F(B+\Delta-s)}{F(B+\Delta-s_L)}. \tag{A-11}$$

If $U(s_L, s_L, B) \leq 0$, then $U(s, s_L, B) \leq 0 < U(s, s, B(s))$, and we are done. So, assume that $U(s_L, s_L, B) > 0$. The first factor on the right-hand side of (A-11) is maximized at $B(s_L) > B$. The second factor is also larger at $B(s_L)$ than at $B < B(s_L)$, because, as explained in Section III, it is increasing in $B$ for $s \geq s_L$. Hence,

$$U(s, s_L, B) \leq U(s, s_L, B(s_L)) \leq U(s, s, B(s)),$$

using the incentive compatibility condition (A-3) for the second inequality.

The proof of (A-5) is symmetric. The dealer's expected profit at a bid $B > B(s_H)$ is:

$$U(s, s_H, B) = U(s_H, s_H, B) \times \frac{F(B+\Delta-s)}{F(B+\Delta-s_H)}. \tag{A-12}$$

As in the previous case, we can assume that $U(s_H, s_H, B) > 0$. The first factor in (A-12) is maximized at $B(s_H) < B$, and the second factor is a decreasing function of $B$ (because $s \leq s_H$); thus,

$$U(s, s_H, B) \leq U(s, s_H, B(s_H)) \leq U(s, s, B(s)).$$

From $x'(s) > 0$, we obtain $B'(s) > 1$. The bid–ask spread is:

$$s + G(x(s)) - B(s) = G(x(s)) + \Delta - x(s),$$

which is strictly decreasing in $s$ because $x$ is strictly increasing in $s$ and $G' < 1$. The probability of trade is $F(x(s))$, which is strictly decreasing in $s$ because $x$ is strictly increasing in $s$ and $F$ is monotone. $\square$

*Proof of Theorem 3.* With some simple algebra, we can write the derivative with respect to $B$ of the objective function in (18) as:

$$f(B+\Delta-s)[A+\Delta-s-h(B+\Delta-s)]. \tag{A-13}$$

This has a unique critical point given by (22). Furthermore, the monotonicity of $h$ (which it inherits from $g$) implies that the derivative (A-13) is positive to the left of the critical point and negative to the right; thus the critical point is the unique maximum. It follows that a solution of (21) is an equilibrium ask.

Trade occurs if and only if $\tilde{v} - \Delta \leq B^o(\tilde{s}) = \tilde{s} + y(\tilde{s}) - \Delta$, so trade occurs if and only if $\tilde{\varepsilon} \leq y(s)$, which occurs with probability $F(y(s))$. We can write (22) as:

$$h(B^o(s) + \Delta - s) = A^o + \Delta - s.$$

Since $h(x) = x + g(x)$, this implies that:

$$B^o(s) + \Delta - s + g(B^o(s) + \Delta - s) = A^o + \Delta - s,$$

which yields a bid–ask spread of:

$$A^o - B^o(s) = g(B^o(s) + \Delta - s) = g(y(s)).$$

It follows that the conditional expected profit of the dealer is $g(y(s))F(y(s))$. The monotonicity conditions stated in the theorem all follow from the monotonicity of $g$ and $F$ and the fact that $y(s)$ is decreasing in $s$. $\square$

## Appendix B. Uniqueness of Equilibrium

Appendix B verifies four claims made in the text.

(I) The D1 criterion rules out pooling equilibria in the transparent market.

(II) There is only one separating equilibrium in the transparent market in which the ask price is $A(s)=R(s,B(s))$ and the bid function is differentiable. That equilibrium is given in Theorem 2.

(III) In any separating equilibrium in the transparent market in which the buyer's beliefs are weakly monotone in the ask price, the ask price must be $A(s)=R(s,B(s))$. Hence, there is only one equilibrium that satisfies the D1 criterion and for which the bid function is differentiable and the buyer's beliefs are weakly monotone in the ask price. That equilibrium is given in Theorem 2.

(IV) In any equilibrium in the opaque market in which the buyer's beliefs are weakly monotone in the ask price, the ask price must be $A=\bar{R}$; that is, the equilibrium ask must be a fixed point of (21).

**(I)** Ramey (1996) shows that the D1 criterion rules out pooling equilibria when a single-crossing property holds. To match our model to Ramey's notation, define

$$V(s,A,B) \quad = \quad (A-B)F(B+\Delta-s).$$

This is the expected profit of a dealer of type $s$ who plays the bid $B$ and ask $A$, when the ask is accepted. In our model, the dealer sets the ask $A$ at the buyer's reservation price, given the buyer's perception of the dealer's type. The reservation price is higher if the perceived dealer type is higher. In Ramey's setup, the bid $B$ is chosen by the sender (the dealer) and the ask $A$ is selected by the receiver (the buyer) after observing the sender's action, with higher asks being chosen by the receiver when the sender's type is thought to be higher. Ramey shows that the D1 criterion rules out pooling equilibria if i) higher actions $A$ are better for the sender (dealer), ii) the action $A$ that would be chosen is increasing in the sender's perceived type, and iii) the ratio

$$\frac{V_3(s,A,B)}{V_2(s,A,B)} \tag{B-1}$$

is increasing in the type $s$, where the subscripts denote partial derivatives (this is the single-crossing property). In our model, $V$ is increasing in $A$ and

$$\frac{V_3(s,A,B)}{V_2(s,A,B)} \quad = \quad -1-\frac{B}{g(B+\Delta-s)}.$$

This is increasing in $s$ because $g$ is an increasing function (due to log concavity). Hence, the D1 criterion rules out pooling equilibria in our model.

To discuss the other items, we need some additional notation. In any equilibrium, the buyer observes the bid and ask and forms beliefs about $\tilde{v}$ and $\tilde{s}$. The beliefs place a probability of 1 on the bid having been accepted, which is the event $B(\tilde{s})\geq\tilde{v}-\Delta$. Let $\mu(\cdot\mid B,A)$ denote the buyer's marginal distribution over $\tilde{s}$ after observing a bid $B$ and ask $A$. The buyer's joint distribution over $(\tilde{s},\tilde{v})$ is determined by $\mu(\cdot\mid B,A)$ and by the exogenously given conditional distribution of $\tilde{v}$, conditioning on $\tilde{s}$ and on the event $B(\tilde{s})\geq\tilde{v}-\Delta$. Assume that the map $A\mapsto\mu(\cdot\mid B,A)$ is continuous in the topology of weak convergence of measures, for each $B$.

The buyer's reservation value after observing a bid $B$ and ask $A$ is:

$$\omega(B,A) \quad \stackrel{\text{def}}{=} \quad \int_{s_L}^{s_H} R(s,B)\,\mu(\mathrm{d}s\mid B,A). \tag{B-2}$$

Because $R(\cdot,B)$ is nondecreasing, $R(s_L,B)\le\omega(B,A)\le R(s_H,B)$ for all $A$. It is optimal for the buyer to accept an ask $A$ if and only if $A\le\omega(B,A)$. Define

$$\text{(B-3)}\qquad \mathcal{A}(B) \quad = \quad \{A\in[R(s_L,B),R(s_H,B)]\mid A\le\omega(B,A)\},$$

and set

$$\text{(B-4)}\qquad \alpha(B) \quad = \quad \sup\mathcal{A}(B).$$

The continuity assumption in the previous paragraph implies that $\omega$ depends continuously on $A$; therefore, $\mathcal{A}(B)$ is compact. Hence, $\alpha(B)\in\mathcal{A}(B)$. The ask $\alpha(B)$ is the maximum ask that will be accepted by the buyer after the dealer plays a bid $B$.

**(II)** For the second item, observe first that the lowest type must play her full information bid $B^\dagger=g^{-1}(\Delta)-\Delta+s_L$ in any separating equilibrium. Suppose otherwise; that is, suppose the lowest type plays some bid $\hat{B}\neq B^\dagger$ in a separating equilibrium. Because the equilibrium is separating, $R(s_L,\hat{B})$ is the buyer's reservation value after observing the bid $\hat{B}$. Hence, the ask $\hat{A}$ played by the lowest type in the separating equilibrium must satisfy $\hat{A}\le R(s_L,\hat{B})$. Therefore, the lowest type dealer's expected profit is:

$$[\hat{A}-\hat{B}]\times F(\hat{B}+\Delta-s_L) \quad \le \quad [R(s,\hat{B})-\hat{B}]\times F(\hat{B}+\Delta-s_L).$$

The definition of the full information optimum $B^\dagger$ is that $B^\dagger$ maximizes the expression on the right-hand side of this inequality. The optimum is unique (see the proof of Theorem 1); thus,

$$[\hat{A}-\hat{B}]\times F(\hat{B}+\Delta-s_L) \quad < \quad [R(s,B^\dagger)-B^\dagger]\times F(B^\dagger+\Delta-s_L).$$

Now, suppose that the lowest type of dealer deviates from $\hat{B}$ to $B^\dagger$. Whatever inference the buyer makes after seeing the deviation, it cannot be worse than the dealer being the lowest type. Therefore, the buyer's reservation price $\alpha(B^\dagger)$ after seeing the deviation cannot be lower than $R(s,B^\dagger)$. Thus,

$$[\hat{A}-\hat{B}]\times F(\hat{B}+\Delta-s_L) \quad < \quad [\alpha(B^\dagger)-B^\dagger]\times F(B^\dagger+\Delta-s_L).$$

However, the right-hand side is the expected profit the lowest type of dealer can achieve after deviating to $B^\dagger$. The inequality shows that the deviation is profitable, contradicting the assumption that the lowest type plays any $\hat{B}\neq B^\dagger$ in a separating equilibrium.

The first-order condition for incentive compatibility is the differential equation (A-7). This must hold in any equilibrium in which the bid function is differentiable. By making the change of variables

$$\text{(B-5)}\qquad x(s) \quad = \quad B(s)+\Delta-s,$$

and using the fact that $G'(x)=[x-G(x)]/g(x)$, we can write the first-order condition (A-7) as:

$$\text{(B-6)}\quad x'(s) \quad = \quad B'(s)-1 \quad = \quad -\frac{U_2(s,s,B(s))}{U_3(s,s,B(s))}-1 \quad = \quad \frac{G(x(s))+\Delta-x(s)}{g(x(s))-\Delta}.$$

The inverse to (B-6) is

$$\text{(B-7)}\qquad s'(x) \quad = \quad \frac{g(x)-\Delta}{G(x)+\Delta-x}.$$

The function (12) is the unique solution of (B-7) satisfying $s(x_L)=s_L$ (note that (B-7) is not actually an ODE, because $s$ does not appear on the right-hand side, so uniqueness follows from the fundamental theorem of calculus, that is, we can compute $s(\cdot)$ just by integrating (B-7)). Therefore, its inverse $x(\cdot)$ is the unique solution of (B-6) satisfying $x(s_L)=x_L=g^{-1}(\Delta)$. Therefore, $B(s)=x(s)-\Delta+s$ is the unique solution of the first-order condition (A-7) satisfying $B(s_L)=g^{-1}(\Delta)-\Delta+s_L$.

**(III)** For the third item, assume that the buyer's beliefs are monotone in the ask price in the sense that the buyer's reservation price $\omega(B,A)$ is weakly increasing in $A$ for each $B$. Whatever bid $B$ any type of dealer plays in any equilibrium, the ask the dealer plays must be $\alpha(B)$, because this is the largest ask the buyer will accept upon learning that there was a transaction at the bid $B$. Thus, for each dealer type $s$, $A(s)=\alpha(B(s))$. We use monotonicity of beliefs to show that $\alpha(B)=\omega(B,\alpha(B))$. Thus, equilibrium asks must be $A(s)=\omega(B(s),A(s))$. In a separating equilibrium, the reservation price of the buyer is $\omega(B(s),A(s))=R(s,B(s))$ by definition. Thus, equilibrium asks are $A(s)=R(s,B(s))$, as claimed. What remains is to show that $\alpha(B)=\omega(B,\alpha(B))$.

To see that $\alpha(B)=\omega(B,\alpha(B))$, note first that, because $\alpha(B)\in\mathcal{A}(B)$, $\alpha(B)\leq\omega(B,\alpha(B))$. Suppose $\alpha(B)<\omega(B,\alpha(B))$. Choose any real number $a$ satisfying $\alpha(B)<a<\omega(B,\alpha(B))$. By monotonicity of beliefs, $a<\omega(B,\alpha(B))\leq\omega(B,a)$, so $a\in\mathcal{A}(B)$. But, then $\alpha(B)<a$ contradicts the fact that $\alpha(B)$ is the supremum of $\mathcal{A}(B)$. This contradiction implies $\alpha(B)=\omega(B,\alpha(B))$.

**(IV)** For the fourth item, let $\hat{\mu}(\cdot\,|\,A)$ denote the buyer's marginal distribution over $\tilde{s}$ and $B$ after observing an ask $A$ and knowing that $\tilde{v}\leq B(\tilde{s})+\Delta$, but without observing $\tilde{s}$ or $B(\tilde{s})$. The buyer's reservation value after observing an ask $A$ is:

$$\text{(B-8)}\qquad \hat{\omega}(A) \quad \stackrel{\text{def}}{=} \quad \int_{[s_L,s_H]\times\mathbb{R}} R(s,B)\,\hat{\mu}(\mathrm{d}s,\mathrm{d}B\mid A).$$

It is optimal for the buyer to accept an ask $A$ if and only if $A\leq\hat{\omega}(A)$. For an equilibrium ask $A^o$, the Bayesian-Nash requirement for a pooling equilibrium specifies that $\hat{\omega}(A^o)=\bar{R}$ defined in (17); that is, the beliefs $\hat{\mu}(\cdot\,|\,A^o)$ must conform to the given exogenous distribution of $\tilde{s}$ and equilibrium play $B^o(s)$.

Define

$$\text{(B-9)}\qquad \hat{\mathcal{A}} \quad = \quad \{A\mid A\leq\hat{\omega}(A)\}$$

and $\hat{\alpha}=\sup\hat{\mathcal{A}}$. As in part (III), the dealer must charge $\hat{\alpha}$ in equilibrium,that is, $A^o=\hat{\alpha}$. Thus, $\hat{\omega}(\hat{\alpha})=\bar{R}$, and we obtain the desired conclusion $A^o=\bar{R}$ once we establish that $\hat{\alpha}=\hat{\omega}(\hat{\alpha})$. This follows as in part (III) when $\hat{\omega}(\cdot)$ is a continuous monotone function.

## Appendix C. Calculations in the Uniform/Normal Model

Appendix C explains how to calculate the equilibria in the transparent and opaque markets in the uniform/normal model. Let N denote the standard normal distribution function, and let n denote the standard normal density function. Then, $F(x)=\mathrm{N}(x/\sigma)$ and $f(x)=\mathrm{n}(x/\sigma)/\sigma$. Hence, $g(x)=\sigma\,\mathrm{N}(x/\sigma)/\,\mathrm{n}(x/\sigma)$. Also,

$$G(x) \quad = \quad -\frac{\sigma\,\mathrm{n}(x/\sigma)}{\mathrm{N}(x/\sigma)} \quad = \quad -\frac{\sigma^2}{g(x)}.$$

Therefore, (12), for the transparent market, becomes:

$$\text{(12}'\text{)}\qquad s(x) \quad = \quad s_L+\int_{x_L}^{x}\frac{g(a)^2-\Delta g(a)}{(\Delta-a)g(a)-\sigma^2}\,\mathrm{d}a.$$

Now, consider the opaque market. Note that

$$\bar{R} \quad = \quad \frac{\mathsf{E}[\tilde{v}1_{\{\tilde{v}\leq B(\tilde{s})+\Delta\}}]}{\mathsf{E}[1_{\{\tilde{v}\leq B(\tilde{s})+\Delta\}}]} \quad = \quad \frac{\mathsf{E}[R(\tilde{s},B(\tilde{s}))F(B(\tilde{s})+\Delta-\tilde{s})]}{\mathsf{E}[F(B(\tilde{s})+\Delta-\tilde{s})]}.$$

The first equality is the definition of the conditional expectation, and the second equality follows from the law of iterated expectations applied to the numerator and denominator separately. Here, we have:

$$R(s,B) \quad = \quad s+G(B+\Delta-s) \quad = \quad s-\frac{\sigma^2}{g(B+\Delta-s)}.$$

Therefore,

$$R(s,B)F(B+\Delta-s) \quad = \quad sF(B+\Delta-s)-\sigma^2 f(B+\Delta-s).$$

Substituting this and (19) into the fixed point condition (21) results in:

$$\begin{aligned} A \quad = \quad & \left(\int_{s_L}^{s_H} F(h^{-1}(A+\Delta-s))\,\mathrm{d}s\right)^{-1} \\ & \times \int_{s_L}^{s_H} [sF(h^{-1}(A+\Delta-s))-\sigma^2 f(h^{-1}(A+\Delta-s))]\mathrm{d}s. \end{aligned}$$

In the integrals, make the change of variable $z=h^{-1}(A+\Delta-s)$. In this normal example, $h'(x)=2+xg(x)/\sigma^2$. Therefore, $\mathrm{d}s=-[2+zg(z)/\sigma^2]\,\mathrm{d}z$ and we obtain

$$\begin{aligned} (21') \qquad A \quad = \quad & \left(\int_{z_H}^{z_L} F(z)\left[2+\frac{zg(z)}{\sigma^2}\right]\mathrm{d}z\right)^{-1} \\ & \times \int_{z_H}^{z_L} [[A+\Delta-z-g(z)]F(z)-\sigma^2 f(z)]\left[2+\frac{zg(z)}{\sigma^2}\right]\mathrm{d}z, \end{aligned}$$

where $z_i=h^{-1}(A+\Delta-s_i)$ for $i=L,H$ (note $z_H<z_L$). Figure 1 plots the right-hand side of (21′) as a function of $A$. The equation means that the equilibrium is at the intersection of this curve with the 45° line. However, the equation can also be simplified to

$$0 \quad = \quad \int_{z_H}^{z_L} [[\Delta-z-g(z)]F(z)-\sigma^2 f(z)]\left[2+\frac{zg(z)}{\sigma^2}\right]\mathrm{d}z.$$

This can be solved for the equilibrium ask $A$ (recall that $z_L$ and $z_H$ depend on $A$).

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